Inference at $*$
of proof for Lemma complete_nat_ind_with_y:

$\vdash \forall P{:}(\mathbb{N} \to \mathbb{P}\{\mathrm{k}\}).\ (\forall i{:}\mathbb{N}.\ (\forall j{:}\mathbb{N} i.\ P(j)) \Rightarrow P(i)) \Rightarrow (\forall i{:}\mathbb{N}.\ P(i))$
by UseWitness $\lambda P,g.\ \mathrm{Y}(\lambda f,x.\ g(x,f))$

1:

$\vdash (\lambda P,g.\ \mathrm{Y}(\lambda f,x.\ g(x,f)))$
$\in (\forall P{:}(\mathbb{N} \to \mathbb{P}\{\mathrm{k}\}).\ (\forall i{:}\mathbb{N}.\ (\forall j{:}\mathbb{N} i.\ P(j)) \Rightarrow P(i)) \Rightarrow (\forall i{:}\mathbb{N}.\ P(i)))$
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http://www.nuprl.org/FDLcontent/p0_942988_/p121_93198_{complete_nat_ind_with_y}.html